MINISTRY OF EDUCATION

NESA NATIONAL EXAMINATION AND SCHOOL INSPECTION AUTHORITY

RTB RWANDA TVET BOARD

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BUGESERA DISTRICT

TSS 2nd TERM DISTRICT COMPREHENSIVE ASSESSMENT
SCHOOL YEAR: 2024-2025

SECTOR: MANUFACTURING AND MINING

……/100 MARKS

TRADE: MAT

RQF LEVEL: 4

MODULE CODE AND TITLE: GENAM401: APPLY FUNDAMENTAL MATHEMATICAL ANALYSIS

DURATION: 3 HOURS

DATE: 17/03/2025

INSTRUCTIONS TO CANDIDATE:

- ✓ **This exam has three sections.**
- ✓ **Maximum marks……/……. Marks**
- ✓ **Attempt all Questions in A**
- ✓ **Choose only two questions in section B.**
- ✓ **Choose only one question in section C.**

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SECTION A: ATTEMPT ALL QUESTIONS /55MARKS

1. Find the domain of definition of the following functions (**L.O.1: Analyze the numerical function**)

a) $\frac{x^2+1}{x^2-x-2}$

Condition: $x^2 - x - 2 \neq 0$ **0.5mark**

Solving this equation by using delta:

$\Delta = b^2 - 4ac$ **0.5mark**

$$1 - 4(-2) = 9$$

$x_{1=2}, x_2 = -1$ **1mark**

Domf=$\mathbb{R}/\{-1, 2\}$ **0.5mark**

b) $f(x) = \sqrt{x^2 - 49}$

condition $\sqrt{x^2 - 49} \geq 0$ **0.5mark**

$x_1 = 7, x_2 = -7$ **0.5mark**

Table of sign

Factors	$-\infty$	**-7**	**7**	$+\infty$
X+7	**- - 0 +**		**+**	
X-7	**- -**	**-**	**0**	**+**
(x-7)(x+7)	**+**	**0 -**	**0**	**+**

1mark

Domf: $]-\infty, -7] \cup [7, +\infty[$ **0.5mark**

2. Show if the following function is even or odd (**l.o.1 Analyze the numerical function**)

$$f(x) = \sqrt{x^2 - 2x + 1}$$

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The function is iff $f(x) = f(-x)$ 1mark

$f(-x) = \sqrt{(-x)^2 - 2(-x) + 1}$ **1mark**

$$f(-x) = \sqrt{x^2 + 2x + 1}$$

$f(x) \neq f(-x)$ **0.5mark**

The function is not even

The function is odd iff $-f(x) = f(-x)$ **1mark**

$-f(x) = -\sqrt{x^2 - 2x + 1}$ **1mark**

The function is not odd 0.5mark

$f(x)$ **is neither odd nor even**

3. What is the range of the function $f(x) = x^2 + 3x + 1$ for the domain=$\{-2, -1, 0, 1, 2\}$ (**L.U.1 Analyze the numerical function)**

The range:

$f(-2) = (-2)^2 + 3(-2) + 1 = -1$ **1mark**

$f(-1) = (-1)^2 + 3(-1) + 1 = -1$ **1mark**

$f(0) = (0)^2 + 3(0) + 1 = 1$ **1mark**

$f(1) = (1)^2 + 3(1) + 1 = 5$ **1mark**

$f(2) = (2)^2 + 3(2) + 1 = 11$ **1mark**

4. Given the function$f(x) = 3x^4 + 7x^3 - 6x^2 - \pi x + 5$. Find $\frac{df(0)}{dx}$ (**L.U.2 Apply fundamental of differentiation)**

$f'(x) = 12x^3 + 21x^2 - 12x - \pi$ **4marks**

$f'(x) = 12(0)^3 + 21(0)^2 - 12.0 - \pi = -\pi$ **1mark**

5. Show that the following function does not have the limit

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$\lim_{x\to 2} |x-5|$

Limit at left:

$\lim_{x\to 2^-} |x-5|$ =- (2-5) =3 **2marks**

Limit at right

$\lim_{x\to 2^+} |x-5|$ =(2-5) =-3 **2marks**

As limit at left ≠ limit at right, this function does not have the limit 1mark (L.U.1 Analyse the numerical function)

6. Lines $x = 2$ and $y = 3$ are asymptotes of $h(x) = \frac{ax+5}{bx+4}$. Calculate the values of **a and b (L.O.1: Analyze the numerical function)**

$\boldsymbol{x = 2\ is\ vertical\ asymptote}$

Then:

$\boldsymbol{b(2)+4=0\ , 2b=-4, b=-2}$ **2.5marks**

$$\boldsymbol{y = 3\ is\ horizontal\ asymptote}$$

Then

$$\lim_{x\to\infty} \frac{ax+5}{-2x+4} = 3$$

By taking the terms with greatest power, we have:

$$\lim_{x\to\infty} \frac{ax}{-2x} = 3$$

$$\frac{a}{-2} = 3$$

$\boldsymbol{a = -6}$ **2.5marks**

7. Find the equation of line passing through the point (2,-3) and perpendicular to the line $y = 4x - 6$ **(L.O.2 Apply fundamental of differentiation)**

Two lines are perpendicular, $\boldsymbol{m_1 m_2 = -1}$ **0.5mark**

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Then $4.m_2 = -1$

$m_2 = -\frac{1}{4}$ **1.5marks**

Equation of line passing this point (2,-3)

$y + 3 = -\frac{1}{4}(x - 2)$ **2.5marks**

8. Calculate the relative asymptotes to the curve

$f(x) = \frac{x^2-2x-8}{x-1}$

V.A$\equiv X = 1$ **1mark**

O.A$\equiv Y = AX + B$ **1mark**

Using long division(or formula):y=x+1 3marks

9. By using Hospital rule, show that $\lim_{x\to\frac{\pi}{2}} \frac{\cos 3x}{\cos x}$ = -3 (**L.O.2 Apply fundamental of differentiation**)

$\lim_{x\to\frac{\pi}{2}} \frac{\cos 3x}{\cos x} = \frac{0}{0}$ $I.F$ **1mark**

Derivative of $cos3x = -3sin3x$ **1mark**

Derivative of $cosx = -sinx$ **1mark**

Then $\lim_{x\to\frac{\pi}{2}} \frac{\cos 3x}{\cos x} = \lim_{x\to\frac{\pi}{2}} \frac{-3sin3x}{-sinx} = \frac{-3sin3(\frac{\pi}{2})}{-\sin(\frac{\pi}{2})} = \frac{3}{-1} = -3$ **2marks**

10. Differentiate the following functions

a) $f(x) = x^3 \cos(x - 1)$

formula: $(uv)' = u'v + v'u$ **0.5mark**

$f'(x) = (x^3)'\cos(x - 1) + (\cos(x - 1))'x^3$ **1mark**

$f'^{(x)} = 3x^2\cos(x - 1) - x^3\sin(x - 1)$ **1mark**

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c) $f(x) = \frac{x}{\sqrt{x}}$

$\boldsymbol{formula}$: $\frac{u}{v} = \frac{u'v - v'u}{v^2} = \frac{x'\sqrt{x} - \sqrt{x})'x}{\sqrt{x)2}} = \frac{\sqrt{x} - \frac{x(x)^{-\frac{1}{2}}}{2}}{x}$ **2.5marks**

11. Calculate the equation of Tangent to the following curve at the given point

$y = 1 - x - 3x^2 - x^3$ at $x = -1$

Equation of line tangent T≡ $y - y_0 = f'(x_0)(x - x_0)$ 0.5mark

$y_0 = 1 + 1 - 3 + 3 = 2$ 1mark

$f'(x) = -1 - 6x - 3x^2$ 0.5mark

$$f'(-1) = -1 - 6(-1) - 3(-1)^2$$

$f'(-1) = -1 + 6 - 3 = 2$ 1mark

T≡ $y - 2 = 2(x + 1)$ 1mark

T≡ $y = 2x + 2 + 2$

T≡ $y = 2x + 4$ 1mark

SECTION B: Choose only three(3) questions /30 marks

12. Determine the following limits:

a) $\lim_{x \to 2} \frac{x - \sqrt{x+2}}{\sqrt{4x+1} - 3} = \frac{0}{0}$ *I.F* **0.5mark**

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$$\lim_{x\to 2}\frac{\frac{x-\sqrt{x+2}}{(\sqrt{4x+1}-3)}\frac{(x+\sqrt{x+2})(\sqrt{4x+1}+3)}{(\sqrt{4x}+3)(x+\sqrt{x+2})}}{} = \lim_{x\to 2}\frac{x^2-(x+2)(\sqrt{4x+1}+3)}{(4x+1-9)(x+\sqrt{x+2})}$$ **1mark**

1mark

$\lim_{x\to 2}\frac{(x+1)(x-2)\sqrt{4x+1}+3}{4(x-2)(x+\sqrt{x+2})} = \frac{3(6)}{4(4)} = \frac{18}{16} = \frac{9}{8}$ **2.5marks**

b) $\lim_{x\to+\infty}\sqrt{x^2+x-2}\ -x\ = \infty-\infty\ \boldsymbol{I.F}$ **0.5mark**

$\lim_{x\to+\infty}\sqrt{x^2+x-2}\ -x = \lim_{x\to+\infty}\frac{\left(\sqrt{x^2+x-2}\ -x\right)(\sqrt{x^2+x-2}\ +x)}{\sqrt{x^2+x-2}\ +x} = \lim_{x\to+\infty}\frac{x^2+x-2-x^2}{\sqrt{x^2+x-2}} = \lim_{x\to+\infty}\frac{x-2}{\sqrt{x^2+x-2}} = \lim_{x\to+\infty}\frac{x}{x} = 1$

4.5marks

c) $\lim_{x\to+\infty}\frac{6x^2+11x+7}{5x^2+2x-1} = \lim_{x\to+\infty}\frac{6x^2}{5x^2} = \frac{6}{5}$ **5marks**

13. Given the function $f(x) = \frac{9}{x^2-9}$. Evaluate:

a) Domain of definition

condition: $x^2-9\neq 0$ **;** $x\neq 3,-3$

Domf=$\mathbb{R}/\{-3,3\}$

b) All possible asymptotes

V.A$\equiv X=-3; X=3$

H.A$\equiv Y=0$

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c) First derivative and its sign

$$f'(x) = \frac{9'(x^2-9) - (x^2-9)'9}{(x^2-9)^2}$$

$$f'(x) = \frac{-18x}{(x^2-9)^2}$$

Table of sign

Factors	$-\infty$	**-3**	**0**	**3**	$+\infty$
-18x		**+ +**	**0**	**---**	
$(x^2-9)^2$		**+ 0**	**+**	**0 +**	
$f'(x)$	**+**	**II**	**+ 0 -**	**II**	**-**
$f(x)$		↗		↘	

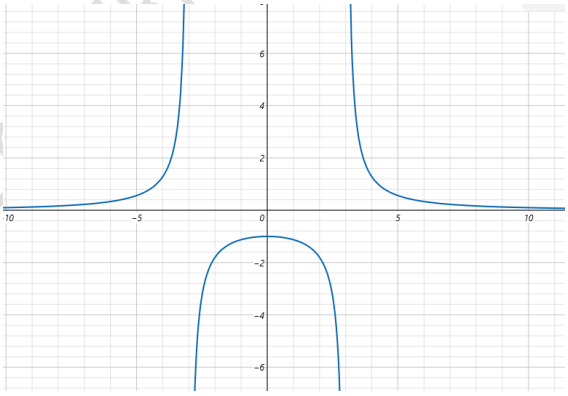
Max(0,-1)

d) Second derivative and its sign

$$f''(x) = \frac{-18(x^2-9)^2 - 2x(x^2-9)(-18x)}{(x^2-9)^4}$$

$$f''(x) = \frac{2x^2+72}{(x^2-9)^3}$$

e) Sketch the graph of $f(x)$.



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14. The function $f(x) = x^3 + \boldsymbol{a}x^2 + \boldsymbol{b}x + 3$ and $\frac{df(x)}{dx}$, each has $x - 1$ as factor.

Determine the value of **a** and **b**. Hence factorize $f(x)$ completely.

$$\boldsymbol{f'(x) = 3x^2 + 2ax + b}$$

$$\boldsymbol{f'(1) = 3(1) + 2a(1) + b}$$

$\boldsymbol{f'(1) = 2a + b = -3}$ **(1)**

$$\boldsymbol{f(1) = 1 + a + b}$$

$\boldsymbol{f(1) = a + b = -1}$**(2)**

Taking equation (1) and(2) and solving it by any method

A=1, b=-5

Then by using synthetic division

The factors are(X-1)(X-1)(X+3)

15. A body moves along the x-axis so that at time seconds $x(t) = t^3 + 3t^2 - 9t$. Evaluate: (**L.O.2 Apply fundamental of differentiation**)

a) The position and velocity at t=1 sec

$$\boldsymbol{x'^{(t)} = v(t) = 3t^2 + 6t - 9}$$

$$\boldsymbol{x(1) = 1 + 3 - 9(1) = -5}$$

$$\boldsymbol{v(1) = 3(1)^2 + 6(1) - 9 = 0}$$

b) Where and when the body comes to rest

The body comes to rest when $\boldsymbol{v(t) = 0}$

$$\boldsymbol{3t^2 + 6t - 9 = 0}$$

Using delta solve this equation to obtain the value of t

$$\Delta = \boldsymbol{36 - 4(3)(-9) = 144}$$

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$t_1 = 1\,second;\ t_2 = -3seconds$

The body comes to rest at t=1second

c) Acceleration of body after 2 second of motion

$$a(t) = v'(t) = 6t + 6$$

$$a(2) = 6(2) + 6 = 18m/s^2$$

16. Calculate the raise the indetermination case: $\lim_{x\to 2} \frac{x^4-4x^3+4x^2}{x^3-3x^2+4} = \frac{0}{0}\ I.F$ (**L.O.2 Apply fundamental of differentiation**)

Using hospital rule:

$$(x^4 - 4x^3 + 4x^2)' = 4x^3 - 12x^2 + 8x$$

$$(x^3 - 3x^2 + 4)' = 3x^2 - 6x$$

$$\lim_{x\to 2} \frac{x^4 - 4x^3 + 4x^2}{x^3 - 3x^2 + 4} = \lim_{x\to 2} \frac{4x^3 - 12x^2 + 8x}{3x^2 - 6x} = \frac{0}{0}\ I.F$$

$$(4x^3 - 12x^2 + 8x)' = 12x^2 - 24x + 8$$

$$(3x^2 - 6x)' = 6x - 6$$

$$\lim_{x\to 2} \frac{4x^3 - 12x^2 + 8x}{3x^2 - 6x} = \lim_{x\to 2} \frac{12x^2 - 24 + 8}{6x - 6} = \frac{12(4) - 24(2) + 8}{6(2) - 6} = \frac{8}{6}$$

SECTION B: Choose only one question

17. 16. $(m-3)x^2 - 8x + 4 = 0$ $\Delta = b^2 - 4ac$.....1mark

$\Delta = 64 - 16(m-3)$.....1mark

$\Delta = -16m + 112$.....2mark

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a) Two real roots $\Delta > 0$ ….0.5mark

$-16m + 112 > 0$ 1mark

$m < 7$ 0.5mark

b) No real root $\Delta < 0$ …0.5mark

$-16m + 112 < 0$ 1mark

$m > 7$ 0.5mark

c) One double root $\Delta = 0$ …0.5mark

$-16m + 112 = 0$ …1mark, $\boldsymbol{m = 7}$ **…0.5mark**

18. Given: $f(x) = 2x^3 - 3x^2 - 12x + 13$ $f'(x) = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$ $= 6(x-2)(x+1)$……2marks

F'(x) =0 if and only if x=-1 or x=2 ……1mark

x	$-\infty$		-1		2		$+\infty$
$f'(x)$	+	+	0	−	0	+	+
	↗		↘		↗		

……5marks

F is increasing if x belongs $]-\infty,-1[\cup]2,+\infty[$……2marks

F is decreasing if x belongs $]-1,2[$ ……2marks

Maximum (-1 ,20) ,Minimum(2,-7).....3marks

Good lucky

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